Linear Programming Bounds of Orthogonal Arrays for Experimental Designs

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***Abstract*—Orthogonal Arrays (OAs) are essential in experimental designs. In this paper, we extend OAs to be more suitable for experimental designs. These are OAs with partial strength. These extended OAs were often considered in applications to experimental designs. Then we propose Linear Programming (LP) bounds for the extended OAs, and show some numerical examples of these LP bounds.**

I. Introduction

Experimental designs in statistics are techniques to get much information from a few experiments [6]. In experimental designs, it is important to construct Orthogonal Arrays (OAs).

OAs are determined by the four parameters; the number of rows $M(\in \mathbb{Z}^+)$, the number of columns $n(\in \mathbb{Z}^+)$, the number of levels q (q is a prime power) and the strength $t(\in \mathbb{Z}^+)$. OAs with the above parameters are described by $OA(M, n, q, t)$. In this paper, we focus on the case $q = 2$, that is $OA(M, n, 2, t)$. Then construction problems for OAs are formulated to find OAs with the minimum M, given n and t. With the problems, it is also important to find lower bounds of M, given n and t.

On the other hand, there are many researches on error-correcting codes [3]. In error-correcting codes, one of the most basic problems is to construct (n, M, d) codes, where n is the code length, M is the number of codewords and d is the minimum distance. Construction problems for binary (n, M, d) codes are formulated to find codes with the maximum M, given n and d, and it is also important to find upper bounds of M, given n and d.

In previous work, some relations between OAs and error-correcting codes were clarified [2],[4],[5],[7],[8]. For example, an $OA(M, n, 2, t)$ can be made from a binary linear (n, M, d) code with the dual distance $t + 1$, where the dual distance means the minimum distance of the dual code. So both results of OAs and error-correcting codes influenced each other.

Delsarte proposed Linear Programming (LP) bounds for OAs and error-correcting codes [1]. Using the LP bounds, a lower bound of M in $OA(M, n, 2, t)$ and an upper bound of M in (n, M, d) codes can be calculated. It was shown that the LP bounds were good bounds from numerical examples.

In the LP bounds, the weight distributions of codes (or OAs) are important parameters. Moreover, two theorems about the weight distribution play important roles. The first is called Macwilliams Theorem. This theorem shows the relation between the weight distributions of dual codes and primary codes. The second is called Delsarte Theorem. This theorem shows the strong constraints of the weight distributions.

In this paper, we extend $OA(M, n, 2, t)$ to OAs with partial strength, $OA(M, n, 2, S)$, $S \subseteq \{0, 1\}^n$, and we propose LP bounds for $OA(M, n, 2, S)$. An $OA(M, n, 2, S)$ is equal to an $OA(M, n, 2, t)$ if $S = \{s \subseteq \{0, 1\}^n | wt(s) \leq t\}$, where $wt(s)$ is the Hamming weight of s. The extended OAs are more suitable for models of experimental designs, which have complicated interaction effects of factors in a experiment. Therefore, these OAs were often considered in applications to experimental designs.

We firstly define $OA(M, n, 2, S)$ and extend the weight distribution to be suitable for $OA(M, n, 2, S)$. Then, we show two theorems about the extended weight distribution, which correspond to Macwilliams Theorem and Delsarte Theorem. And, we lead to LP bounds of $OA(M, n, 2, S)$ using the two theorems. Lastly, we show some numerical examples of the LP bounds.

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